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Analytical Longitudinal Speed Planning For CAVs with Previewed Road Geometry and Friction Constraints\*

*Abstract*— Due to the lack of information, current vehicle control systems generally assume that the friction ahead of a vehicle is unchanged relative to the vehicle's position. However, with connectivity either to other vehicles, infrastructure, or cloud services, future vehicles may have access to this information which is particularly valuable for planning velocity trajectories ahead. This work introduces a method for planning longitudinal speed profiles for Connected and Autonomous Vehicles (CAVs) that have previewed information about road geometry and friction conditions. The novelty of this approach is to explicitly include consideration of the friction ellipse available to the vehicle to develop an analytical solution to the allowable velocity profile that prevents departure from the friction ellipse. The results further define the relationship between the minimum preview distance and longitudinal velocity limits that ensure the vehicle has sufficient time to take action for upcoming hazardous situations. The efficacy of the algorithm is demonstrated through an application case where a vehicle is navigating curvy roads with changing friction conditions at maximum speeds, with results showing that the vehicle consistently operates within the available friction limits.

# INTRODUCTION

The traffic accident is fatal and has a huge negative influence on economic costs. [follow some data from reference] Many factors attribute to safety, but a large part of the accidents are caused by slick roads (snow, ice, raining, etc.) and overspeeding. (make it positive: maximize the mobility throughout the highway system in any condition)

One way to make cars safer is the development of the ADASs (Advanced Driver Assistance Systems) such as EBS, LKA, and ESC, etc., which can assist the driver to follow the desired path stably in hazardous situations. The assistance system can be categorized into lateral control, longitudinal control, and hybrid control. The controller can output proper steering angle and/or driving/braking torque command once the vehicle is out of stability. For example, Yu, et al. [1] designed a feedback-feedforward steering controller to improve the vehicle stability when hard-braking maneuvers on road with split friction. Li, et al. presented a torque control strategy for the situation of abrupt changes in the road friction [2].

However, most of the controllers are activated only when the vehicle states have a significant deviation from the nominal value. Additionally, the controller has a stable region. Consequently, the controller even fails to keep the vehicle stable if the road condition changes intensively. [ref]

Different from the reaction controller, researchers developed the proactive envelope controller with real-time road friction estimation, which can keep the vehicle within a stable region. But it still can not certainly prevent the vehicle from veering away from the desired path when the road condition changes intensively.

Existing research has shown that appropriate longitudinal velocity planning is vital when following a path with tight curvature change [3]. The idea is to reduce vehicle speed before a potentially dangerous situation is reached, in contrast with widely used stability control systems that only react once loos of control is imminent. The insight is that the vehicle could have a larger stable operating region to follow the desired path with a smaller velocity [4][5].

However, they only consider the variation of curvature for longitudinal planning, and little attention has been paid to the road surface friction condition changing when conducting longitudinal velocity planning. For example, if the vehicle can preview the path friction reduction and slow down appropriately before entering the slick region, then a complicated algorithm is not necessary to stabilize the vehicle. The prior estimation of friction and peak tire force, before the slick region is reached, allows a vehicle chassis control system to work more reliably and proactively [2]. As a result, even a normal steering control algorithm can enable the vehicle to follow the path well.

Unfortunately, it is almost impossible to preview the large area road condition just through the intelligence of individual vehicle, where each vehicle itself constantly measuring and navigating the world using in-vehicle system. A potential solution to preview the road surface friction condition is motivated by the increasing research into the cyber-physical system, especially Connected and Autonomous Vehicles (CAV). This solution takes advantage of network intelligence instead of individual intelligence, where the pre-measured road friction information from the individual vehicle is shared within the network. Within the network, the information can flow between vehicle to vehicle, vehicle to infrastructure, and vehicle to database systems. In this way, each vehicle and the roadside unit can measure local road friction [6][7] and share it with the cloud database. Each vehicle in the network can query the aggregated friction information from the shared database. A demo framework of this network is shown in Fig. 1.

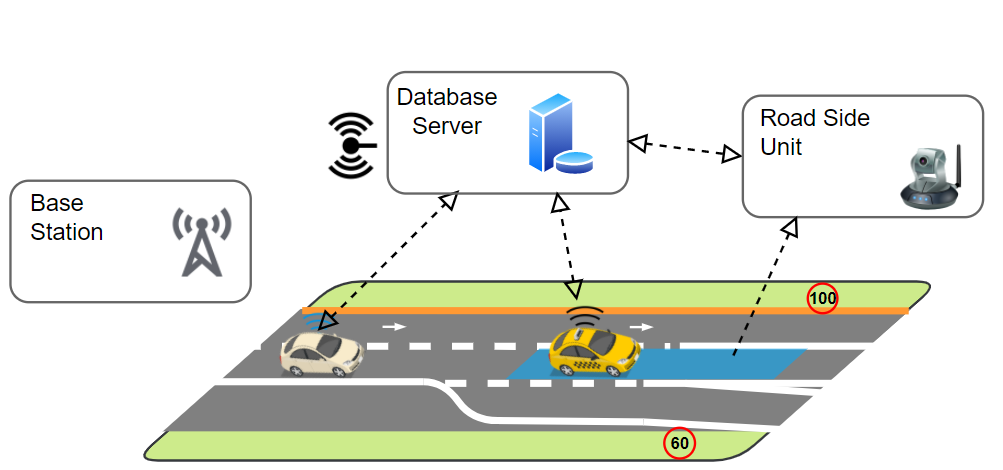


Figure 1. A strategy of road friction preview with a roadside shared roadway database. (need a better diagram to show the data sharing idea) The database can leverage shared intelligence to substantially improve the operation of each individual in the population.

The improvements in vehicle safety can be achieved by limiting the vehicle speed based on the road condition. In this paper, we introduce an idea of road friction preview through database-informed CAV. Then inspired by the work in DDL on vehicle dynamics at the limits of handing [8]–[10], we present an algorithm to generate a longitudinal vehicle velocity limit profile for a given desired path with the preview of path friction. Moreover, relationships are established between data confidence, preview distance, and vehicle speed for a vehicle traversing a roadway system augmented with such a preview system.

The remainder of this paper is organized as follows: Section II discusses the velocity limit profile planning based on the tire limits. Section III analyzes the preview distance. Section IV shows an application case. Finally, a conclusion section summarizes the main results of the work.

# Vehicle Longitudinal Velocity Planning Given a Reference Path

This section presents the generation of limit speed profile which vehicles can achieve without exceeding available tire friction limits constraints [11]. At first, the longitudinal dynamic driving on the friction circle is derived, and then introduce the approach to describe the path by station s, curvature κand friction coefficient. Finally, show the detailed velocity planning method.

## A. Vehicle Chassis Model and Tire Friction Limits

The vehicle dynamic equations for the three states single track model shown in Fig. 2 are:







where longitudinal velocity *Ux*, lateral velocity *Vy* and yaw rate *r* are the three states. *θ* is the path grade, and *g* is the gravitational acceleration. The vehicle parameters include the vehicle’s mass *m*, yaw moment of inertia *Izz*, front wheel steering angle δ, and a and b the distance from the vehicle’s center of gravity to the front and rear axle respectively. Forces *Fxf, Fyf, Fxr, Fyr* are the forces acting on the front and rear tires.



Figure 2. Planar single-tack vehicle chasis model (need a high resolution version)

The available longitudinal force *Fx* and lateral force *Fy* at each tire is constrained by friction circle:





where μ is the road-tire friction coefficient, and *Fzf* and *Fzr* are the normal force at the front and rear axle respectively. If we ignore the load transfer, the normal forces are, .

Determining the limit speed profile requires the vehicle to utilize all the available tire friction to generate forces so that vehicle can operate at acceleration limits to achieve the maximum safe speed [12]. It implies that all tire forces need to remain on the boundary of the friction circle:





Fig. 3 shows the tire force when maneuvering through a left corner of a path. The boundary of the friction circle depends on the road-tire friction coefficient.

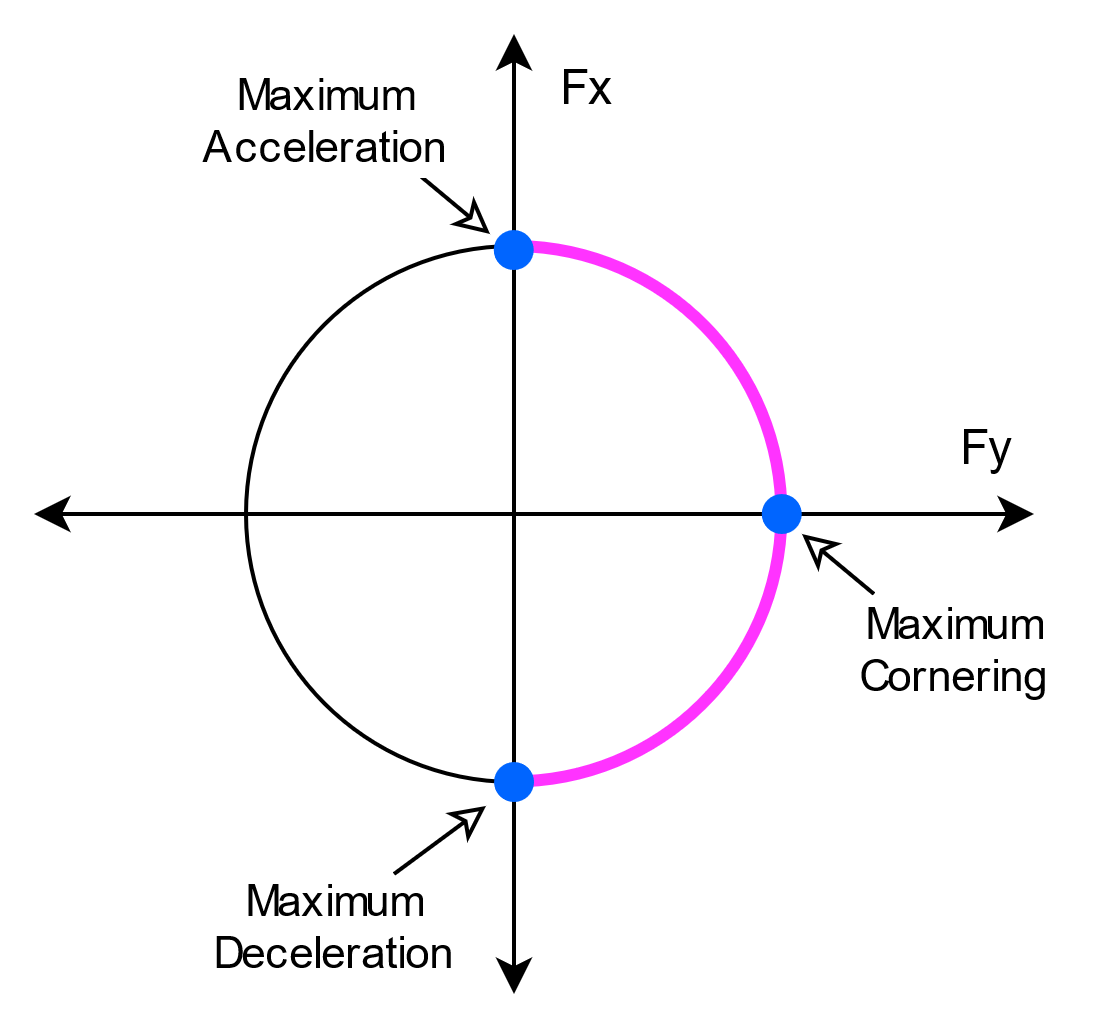


Figure 3. The maximum performance of vehicle can be achieved by driving at the boundary of friction circle. This plot shows the left cornering case.

As we are interested in the longitudinal dynamic, assume the lateral states are steady:



where κ is the path curvature.

Substituting , , and into , , and yields:



Before going further, we add an adjusting parameter λ into the equation for two reasons: a. compensation for the uncertainty of friction preview; b. common driver may not able to operate a vehicle at friction limit as racecar drivers or autonomous driving systems [13]. We have the dynamics equation that depicts the maximum available longitudinal acceleration:



where the plus-minus sign (±) corresponds to acceleration and deceleration respectively. Positive *θ* is for upgrades and negative is for downgrades.The parameters μ and κ in depend on the path position. Thus we introduce the path description method in the following session.

## B. Path Representation

This paper doesn’t focus on path planning. Therefore, the desired path is assumed to be given. The clothoid path description is widely used for highway road design [14] and vehicle path planning, for example, the racing line [10], [15] and minimum curvature optimal path [16] [17]. A clothoid path can generally be described by a succession of turns - consist of spirals and constant radius arcs - and straight lines. The curvature of the spiral is linearly increasing along with the distance:



where *s* called “station” in this paper is the distance measured along a path; *Ls* is the total length of the spiral and κc is the curvature at the end of the spiral. In this way, the curvature of the whole path can be described by a succession of linear functions.

Without loss of generality, an example oval path similar to the Larson Institute Test Track [18] is shown in Fig. 4. The path is given according to the highway road design rule which decomposes the cornering into three phases: an entry clothoid ~~for trail braking~~, a circle arc ~~for pure cornering~~, and an exit clothoid ~~for throttle exit~~ [10]. The curvature and previewed friction coefficient and grade are shown in Fig. 5. ~~The friction can be the minimum friction at each tire for the friction split case.~~ All of these path parameters can be described as a function of the path station: κ(s), μ(s), θ(s). The example path where the grade is not zero is similar to the off-ramp and on-ramp part of a highway.



Figure 4. A circular oval smaple path. (update to a real oval).



Figure 5. The curvature, previewed friction coefficient, and previewed grade for sample path. We assume a abrupt friction changing.

## C. Velocity Profile Generation

Velocity planning has a significant impact on driving safety, especially when vehicles drive on a road with changing friction and geometry. With the longitudinal dynamic equation (9) and the desired path description, the speed profile can be determined. The approach presented in this paper is inspired by those works: three passes [11], nonlinear optimization [3], segment and iteration [19], where a velocity profile is planned given the path curvature.

Express the longitudinal acceleration with respect to the station:



Substitute into yields:



can be solved using a numerical integration method as we can not find a general analytical solution for all cases.



where . The solution will be accurate enough if path waypoints are dense, i.e. the is small enough. In this paper, we choose  smaller than 0.1m. λ=0.95 is taken.

The first step of generating the speed profile is to find the maximum permissible steady-state vehicle velocity with zero longitudinal acceleration, which is given by :



Notice that the steady-state speed will be very high when the curvature is small (the curvature is zero for a straight line) and thus a speed limit of 60m/s was imposed. The first step result is shown as the “curve limit speed” in Fig. 6.

The following next step is a forward integral step:



It starts from the vehicle's current speed. At each step, the result is compared to the curve limit speed, and the minimum value is taken. This step indicates how fast a vehicle can accelerate.

The final step is a backward integral step:



It starts from the maximum allowable vehicle speed at the end of the path and back toward to current station. At each step, the result is compared to the forward integral results. This step indicates how fast a vehicle can decelerate. The speed profile results and the acceleration for the example path are shown in Fig. 6. Due to the adjusting parameter λ, the acceleration of the planned speed profile can not reach the maximum available acceleration.





Figure 6. (a) The intermediate and final results of the speed profile of the computation algorithm. (b) The acceleration.

# Preview Distance

When a vehicle query friction preview from a shared database, a tradeoff problem is to determine a proper preview distance. The longer preview distance involves more data transmitting which results in more time delay and data cost, but with a shorter preview distance, a vehicle could not have enough space to respond to the dangerous situation ahead. Therefore, this section presents a way to determine the minimum friction preview distance so that it can have sufficient time to take action for upcoming hazardous situations.

## A. Minimum Friction Preview Distance

The criterion for determining the preview distance in this paper is that a vehicle always has enough space to stop within the distance. The strategy is to calculate the stop distance with the most critical scenario for conservation. Therefore, we assume that the vehicle is driving on a snowy road with a constant road-tire friction coefficient value, i.e. λμ=0.2 and with a maximum permissible initial speed. With this assumption, our task is to find the scenario where the vehicle stop with the longest distance.

~~Section II.B indicates that a path comprises straight lines with zero curvature, radius arcs with constant curvature, and spirals with linear curvature~~. From section II, for a given path, the straight-line segment could allow the most permissible initial speed. And according to , the arc path has the minimum deceleration and thereby maximum stop distance( the larger the curvature, the smaller the most permissible initial speed. And for a given initial speed, a larger curvature road has a longer stop distance). Thus, the most critical scenario is that a vehicle driving at the speed limit Ux0 at a straight-line road, then enter a spiral road, and finally stop at the arc segment, which is shown as Fig. 7.



Figure 7. The scenario where a vehicle decelerates starts from a line segment with an initial speed Ux0.

Now the task is to analyze the deceleration behavior at the segment sequence.

For the line segment path whose curvature is zero, the stop distance can be determined by solving analytically:



The stop distance increases quadratically with the initial speed *Ux0*. And

For the spiral path, we suppose its length is:



Then the numerical solution of the station can be derived from :





for *k = 0,1,2,3,…, N*, where , and *s0=0*, *Ux(s0) = Ux\_lf* and Ux(sN) = Ux\_a0= sqrt(μg/κc)

For the radius arc path whose curvature is a constant, the stop distance can be determined by solving analytically:



implies that the stop distance is increasing with the increase of *Ux0*, but *ds\_a* has a supremum:



The supremum is independent of friction μ, and the reason is that the initial speed on the circular arc is limited by curvature and friction.

The total stop distance is the addition of , and :



On the entering clothoid path, for a fixed initial speed, using the numerical solution of (12) we can find the stop distance will be longer for lower friction and larger curvature slope. And the stop distance is much large than case a with the same initial speed. The stop distance is approximately 350m for a 35m/s initial speed and friction coefficient of 0.2.

The minimum curvature can be obtained from the path, and the minimum friction can be assumed to be 0.1 for conservation.

The road is designed with good friction, refer to the 3.2.2.2 [1] AASHTO, *A Policy on Geometric Design of Highways and Streets*. 2018.



Figure 7. The stop distance on entering clothoid path with an initial spped of 50m/s.

[more plots: stop distance vs intial speed ]

The worst case is moving at the clothoid segment, ~~entering a circle,~~ with a sudden friction decrease.

Based on previous analysis, a road to is the On-ramp and off-ramp scenario. For off-ramp road, a vehicle needs to slow down appropriately from a straight road to a spiral and circle

Considering the delay of the driver, vehicle systems, and data transmission. The preview distance is a function of *Ux*, *ax*, max(κ), assumed min μ, and constant.



Most human drivers with the required driving vision can only see approximately XXXm even on a clear day when driving. Unfortunately, the visibility reduces a lot at adverse weather where dangerous road condition occurs.

And the driver’s sight is limited at path corners and hills.

The friction preview is vital for safety.

# Application Case and Simulation Results

A vehicle with rear driving, front steering, understeering, or oversteering.

A path with a length of 5 km, compares the velocity results considering only curvature and the results considering both friction and curvature.

A simple driver controller (look ahead error method which is similar to the human driver may need to find a reference) can follow the path well with the preview of friction.



Figure 9. The velocity profiel with only curvatrue(assue the friction coefficient is 0.9 and the grade is 0), and with the preview of grade and friction

A controller behavior with limited friction preview distance.



Figure 5. The lateral error betwee the preview

# Conclusion and Future Work

In this paper, we . The idea behind this is to extend individual intelligence with network intelligence.

For the preview distance, this paper just presents the most conservative results.

## Speed Profile Depends on the Preview Distance

The vehicle needs to move slower if with less visibility.

Both road condition and preview distance influence the vehicle speed profile.

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